

Last Name	First Name	Student ID Number
Solution		

Prob #	1	2	3	4	Total
Points	21	29	25	25	

Time: 80 Minutes

Seat Number:

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$$F(\mathbf{x}) = F(\mathbf{x}^*) + \nabla F(\mathbf{x})^T \Big|_{\mathbf{x} = \mathbf{x}^*} (\mathbf{x} - \mathbf{x}^*) \\ + \frac{1}{2} (\mathbf{x} - \mathbf{x}^*)^T \nabla^2 F(\mathbf{x}) \Big|_{\mathbf{x} = \mathbf{x}^*} (\mathbf{x} - \mathbf{x}^*) + \dots$$

$$\frac{\mathbf{p}^T \nabla F(\mathbf{x})}{\|\mathbf{p}\|} \quad \frac{\mathbf{p}^T \nabla^2 F(\mathbf{x}) \mathbf{p}}{\|\mathbf{p}\|^2} \quad \alpha_k = -\frac{\mathbf{g}_k^T \mathbf{p}_k}{\mathbf{p}_k^T \mathbf{A} \mathbf{p}_k}$$

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \alpha_k \mathbf{g}_k \quad \mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_k \mathbf{p}_k$$

$$L_i = \sum_{j \neq i} \max(0, y_j - y_i + \Delta)$$

$$S(y_i) = \frac{e^{y_i}}{\sum_j e^{y_j}}$$

$$H(p, q) = -\sum_x p(x) \log(q(x))$$

$$L_i = -\log\left(\frac{e^{y_i}}{\sum_j e^{y_j}}\right)$$

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1. Consider a convolutional neural network.

Note: **Do NOT consider Biases.**

Input layer:

Input to this CNN are color images of size **100x70x3** with the **batch size = 30**

Note: Input image has **different horizontal and vertical** resolution.

Next layer is Conv2D layer:

Number of filters: **15**, filter size: **9x9**; stride: **3x3**; padding: **4x4**

Q1: What is the shape of the weight matrix for this layer?

Q1: 9x9x3x15

Q2: What is the shape of the output (tensor) of this layer?

Q2: 30x34x24x15

Next layer is Conv2D layer:

Number of filters: **10**, filter size: **4x4**; stride: **2x2**; padding: **1x1**

Q3: What is the shape of the weight matrix for this layer?

Q3: 4x4x15x10

Q4: What is the shape of the output (tensor) of this layer?

Q4: 30x17x12x10

Next layer is Flatten layer:

Q5: What is the shape of the output (tensor) for this layer?

Q5: 30x2040

Next layer is Dense layer:

Number of nodes: **50**

Q6: What is the shape of the weight matrix for this layer?

Q6: 2040x50

Q7: What is the shape of the output (tensor) for this layer?

Q7: 30x50

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Problem 1 Continued

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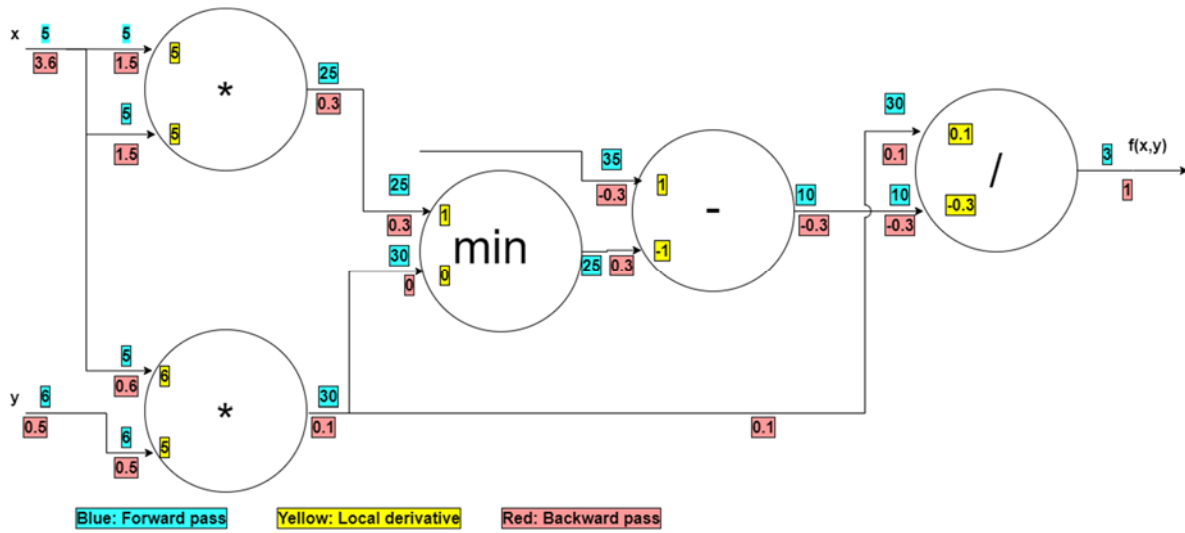
2. Consider the expression:

$$f(x, y) = \frac{xy}{35 - \min(xy, x^2)}$$

given the inputs: $x = 5, y = 6$

Draw the computational graph and calculate the $\frac{\delta f(x,y)}{\delta x}$ and $\frac{\delta f(x,y)}{\delta y}$

For proper credit, you **MUST SHOW** all the numerical values **for each node** as they flow in the forward and backward path in the computational graph.



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Problem 2 Continued

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3. Using tensorflow, complete the following function to create and train a **two-layer** neural network. The **first layer** has **7 sigmoid** nodes. The **output layer** has **linear nodes**. Loss function should be **MSE**. Anything not specified in the description should be **inferred from the function's parameters and not hardcoded**.

Code should include initializing weights, training loop with forward pass, gradient calculation, and weight updates.

You may assume the entire dataset is one batch.

DO NOT USE Keras

```
import numpy as np
import tensorflow as tf
def create_and_train_nn(X, Y, epochs, alpha):
    """
    :param X: Array of input [n_samples, input_dimensions]
    :param y: Array of desired outputs [n_samples , target_dimension].
    :param epochs: number of epochs
    :param alpha: Learning rate:
    :return w1, w2 Weight matrices."""

    w1=tf.Variable(np.random.randn((X.shape[1],7)))
    b1=tf.Variable(np.random.randn((7)))
    w2=tf.Variable(np.zeros((7,Y.shape[1])))
    b2=tf.Variable(np.random.randn((Y.shape[1])))

    for epoch in range(epochs):
        with tf.GradientTape() as tape:
            y1=tf.sigmoid(tf.matmul(X,w1)+b1)
            y2=tf.matmul(y1,w2)+b2
            loss=tf.reduce_mean(tf.square(Y-y2))
            dw1,dw2,db1,db2=tape.gradient(loss,[w1,w2,b1,b2])
        w1.assign_sub(alpha*dw1)
        w2.assign_sub(alpha*dw2)
        b1.assign_sub(alpha*dw1)
        b2.assign_sub(alpha*dw2)

    return w1,w2
```


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4. Complete the code for the following function.

USE numpy only. DO NOT USE tensorflow or keras

```
import numpy as np
def calculate_svm(yhat, yt):
    # This function calculates the SVM error for the entire data set
    # yhat: Array of actual outputs [num_of_samples, num_of_classes]
    # yt: Array of desired outputs [num_of_samples]
    # Each element of yt array is the index of the true class.
    # return: SVM for the entire data set (A single float number).
    # Return value is the average of all the SVMs for the samples.
    # Assume delta is equal to 1
```

```
    # Detail solution
    number_of_samples=yhat.shape[0]
    number_of_classes=yhat.shape[1]
    total_loss=0
    for sample_index in range(number_of_samples):
        target_class_index=yt[sample_index]
        yi=yhat[sample_index, yt[sample_index]]
        sample_loss=0
        for class_index in range(number_of_classes):
            if target_class_index==class_index:
                continue
            yj=yhat[sample_index, class_index]
            sample_loss=sample_loss+np.maximum(0, yj-yi+1)
        total_loss=total_loss+sample_loss
    total_loss=total_loss/number_of_samples
    return total_loss
```

```
def calculate_svm_v2(yhat, yt):
    # More compact solution
    total_loss = 0
    for k in range(yt.shape[0]):
        loss = np.maximum(0, yhat[k] - yhat[k][yt[k]]+1)
        loss[yt[k]] = 0
        total_loss=total_loss+np.sum(loss)
    return total_loss / yhat.shape[0]
```

```
def calculate_svm_v3(yhat, yt):
    # Another solution
    total_loss = 0
    for sample, target_index in zip(yhat, yt):
        margins = np.maximum(0, sample - sample[target_index] + 1)
        total_loss = total_loss + np.sum(margins) - 1
    return total_loss / yhat.shape[0]
```

```
def calculate_svm_v4(yhat, yt):
    # Most compact solution with numpy
    c=yhat.shape[0]
    return (np.sum(np.maximum(0, yhat-[yhat[np.arange(c)[:None], yt.reshape(c,1)]] + 1))-c)/c
```
